Sets and Subsets

<u>Set</u> - A collection of objects. The specific objects within the set are called the *<u>elements</u>* or *<u>members</u>* of the set. *Capital letters* are commonly used to name sets.

Examples: $Set\ A = \{a, b, c, d\}$ $\quad or \quad$ $Set\ B = \{1, 2, 3, 4\}$

<u>Set Notation</u> - Braces { } can be used to list the members of a set, with each member separated by a comma. This is called the "<u>Roster Method</u>." A description can also be used in the braces. This is called "<u>Set-builder</u>" notation.

Example: *Set A:* The natural numbers from 1 to 10.

Members of A: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ← **Roster Method**

Set Notation: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set Builder Not.: $\{x \,|\, x\ is\ a\ natural\ number\ from\ 1\ to\ 10\}$

<u>Ellipsis</u> - Three dots (…) used within the braces to indicate that the list continues in the established pattern. This is helpful notation to use for *long lists* or *infinite lists*. If the dots come at the end of the list, they indicate that the list goes on indefinitely (i.e. an infinite set).

Examples: *Set A:* Lowercase letters of the English alphabet

Set Notation: $\{a, b, c, \ldots, z\}$

<u>Cardinality of a Set</u> – The number of *distinct* elements in a set.

Example: Set A: The days of the week

Members of Set A: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

Cardinality of Set $A = \boldsymbol{n(A)} = 7$

<u>Equal Sets</u> – Two sets that contain exactly the same elements, regardless of the order listed or possible repetition of elements.

Example: $A = \{1, 1, 2, 3, 4\}$, $B = \{4, 3, 2, 1, 2, 3, 4,\}$.

Sets $A\ and\ B$ are equal because they contain exactly the same elements (i.e. 1, 2, 3, & 4). This can be written as $\boldsymbol{A = B}$.

Equivalent Sets – Two sets that contain the same number of distinct elements.

Example: $A = \{Football, Basketball, Baseball, Soccer\}$
$B = \{penny, nickel, dime, quarter\}$

Both Sets have 4 elements

$n(A) = 4 \quad and \quad n(B) = 4$

$A\ and\ B\ are\ Equivalent\ Sets,\ meaning\ n(A) = n(B).$

Note: If two sets are Equal, they are also Equivalent!

Example: $Set\ A = \{a, b, c, d\}$ $\quad Set\ B = \{d, d, c, c, b, b, a, a\}$

Are Sets A and B Equal?

Sets A and B have the exact same elements! $\{a, b, c, d\}$

→ **Yes!**

Are Sets A and B Equivalent?

Sets A and B have the exact same number of distinct elements! $n(A) = n(B) = 4$

→ **Yes!**

The Empty Set ***(or Null Set)*** – The set that contains **no elements**.
It can be represented by either $\{\ \ \}$ *or* $\emptyset$.

Note: Writing the empty set as $\{\emptyset\}$ is **not correct**!

Symbols commonly used with Sets –

$\in \rightarrow$ *indicates an object is an* ***element*** *of a set.*

$\notin \rightarrow$ *indicates an object is* ***not*** *an element of a set.*

$\subseteq \rightarrow$ *indicates a set is a* ***subset*** *of another set.*

$\subset \rightarrow$ *indicates a set is a* ***proper subset*** *of another set.*

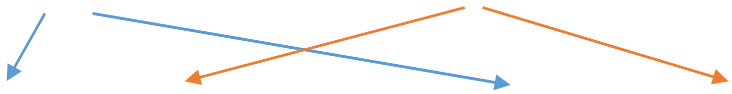
$\cap \rightarrow$ *indicates the* ***intersection*** *of sets.*

$\cup \rightarrow$ *indicates the* ***union*** *of sets.*

Subsets - For Sets *A* and *B*, Set *A* is a **Subset** of Set *B* if every element in Set *A* is also in Set *B*. It is written as $\boldsymbol{A} \subseteq \boldsymbol{B}$.

Proper Subsets - For Sets *A* and *B*, Set *A* is a **Proper Subset** of Set *B* if every element in Set *A* is also in Set *B*, ***but** Set A does **not** equal Set B.* $(\boldsymbol{A} \neq \boldsymbol{B})$ It is written as $\boldsymbol{A} \subset \boldsymbol{B}$.

Example: $Set\ A = \{2, 4, 6\}$ $Set\ B = \{0, 2, 4, 6, 8\}$



$\{2, 4, 6\} \subseteq \{0, 2, 4, 6, 8\}$ **and** $\{2, 4, 6\} \subset \{0, 2, 4, 6, 8\}$

Set *A* is a **Subset** of Set *B* because every element in *A* is also in *B*. $\boldsymbol{A} \subseteq \boldsymbol{B}$

Set *A* is a **Proper Subset** of Set *B* because every element in *A* is also in B, but A ≠ *B*. $\boldsymbol{A} \subset \boldsymbol{B}$

Note: *The Empty Set is a Subset of every Set.*
The Empty Set is also a Proper Subset of every Set except the Empty Set.

Number of Subsets – The number of distinct subsets of a set containing n elements is given by $\mathbf{2^n}$.

Number of Proper Subsets – The number of distinct proper subsets of a set containing n elements is given by $\mathbf{2^n - 1}$.

Example: How many Subsets and Proper Subsets does Set *A* have?

$Set\ A = \{bananas, oranges, strawberries\}$
$n = 3$

Subsets = $2^n = 2^3 = 8$ **Proper Subsets** = $2^n - 1 = 7$

Example: List the **Proper Subsets** for the Example above.

1. $\{bananas\}$
2. $\{oranges\}$
3. $\{strawberries\}$
4. $\{bananas, oranges\}$
5. $\{bananas, strawberries\}$
6. $\{oranges, strawberries\}$
7. $\emptyset$

Intersection of Sets – The Intersection of Sets *A* and *B* is the set of elements that are in both *A* and *B*, *i.e. what they have in common*. It can be written as $\boldsymbol{A} \cap \boldsymbol{B}$.

Union of Sets – The Union of Sets *A* and *B* is the set of elements that are members of Set *A*, Set *B*, or both Sets. It can be written as $\boldsymbol{A} \cup \boldsymbol{B}$.

Example: Find the Intersection and the Union for the Sets *A* and *B*.

$Set\ A = \{Red, Blue, Green\}$

$Set\ B = \{Yellow, Orange, Red, Purple, Green\}$

Intersection: $\boldsymbol{A} \cap \boldsymbol{B} = \{Red, Green\}$

Set *A* and *B* only have 2 elements in common.

Union: $\boldsymbol{A} \cup \boldsymbol{B} = \{Red, Blue, Green, Yellow, Orange, Purple\}$

List each distinct element only once, even if it appears in both Set A and Set B.

Complement of a Set - The Complement of Set *A*, written as ***A'***, is the set of all elements in the given Universal Set (*U*), that are not in Set *A*.

Example: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1,3,5,7,9\}$

Find A'.

Cross off everything in U that is also in A. What is left over will be the elements that are in A'

$U = \{\cancel{1}, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, 8, \cancel{9}, 10\}$

So, $A' = \{2, 4, 6, 8, 10\}$

Try these on your own!

Given the set descriptions below, answer the following questions.

$U = All\ Integers\ from\ 1\ to\ 10.$ $A = Odd\ Integers\ from\ 1\ to\ 10,$

$B = Even\ Integers\ from\ 1\ to\ 10,$ $C = Multiples\ of\ 2\ from\ 1\ to\ 10.$

1. Write each of the sets in roster notation. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{2, 4, 6, 8, 10\}$
2. What is the *cardinality* of Sets U and *A*? *Cardinality: U → 10, A → 5*
3. Are Set *B* and Set *C* *Equal*? *Yes, they are Equal*
4. Are Set *A* and Set *C* *Equivalent*? *Yes, they are Equivalent*
5. How many *Proper Subsets* of Set U are there? $2^{10} - 1 = 1023$
6. Find $\boldsymbol{B}'$ *and* $\boldsymbol{C}'$ $B' = C' = \{1, 3, 5, 7, 9\}$
7. Find $\boldsymbol{A} \cup \boldsymbol{C}'$ $A \cup C' = \{1, 3, 5, 7, 9\}$
8. Find $\boldsymbol{B}' \cap \boldsymbol{C}$ $B' \cap C = \{\ \}$ or $\emptyset$